

New $\mathcal{N} = 1$ Superconformal Field Theories In Four Dimensions

Ibrahima Bah¹ and Brian Wecht^{1,2}

¹ Michigan Center for Theoretical Physics

University of Michigan

Ann Arbor, MI 48109 USA

² Center for the Fundamental Laws of Nature

Harvard University

Cambridge, MA 02138 USA

We construct a large new family of four-dimensional $\mathcal{N} = 1$ superconformal field theories by coupling Gaiotto's T_N theories to $\mathcal{N} = 1$ vector multiplets. In particular, we consider theories in which various T_N blocks are linked together via bifundamental and adjoint chiral superfields, with no superpotential. We find that while some of these constructions appear to give new strongly coupled SCFTs, others lead to violations of the a -theorem, and thus do not appear to be good interacting theories.

1. Introduction

The infrared phases of four-dimensional quantum field theories are notoriously difficult to study. Even if a theory starts off with a weakly coupled description in the ultraviolet, it can (and often does) then flow to some strongly coupled phase at long distances. The particular details of the infrared phase are often not at all obvious from the free UV description, and even when we know the answer (as in QCD), the underlying dynamics can still be wildly out of theoretical control. We thus often think of the UV Lagrangian as something that is given to us, and relegate our ignorance to the mysteries of the infrared.

It has been known for some time that it is occasionally possible to make detailed statements about quantum field theories even when a weakly coupled UV Lagrangian is not available. When the theory is in a conformal phase, the constraints provided by the additional symmetries allow for exact results which are often not accessible in non-conformal theories. This increased power makes a Lagrangian useful but not strictly necessary, depending on the questions in which one is interested. For example, there are well-known such cases in two dimensions (e.g. minimal models) where we can say many things without any reference to a Lagrangian.

Four-dimensional CFTs are not nearly as constrained as those in two dimensions, since the conformal algebra no longer has infinite dimension. However, there are still many theories in which we can make progress without a weakly coupled description, as long as we have enough supersymmetry. The most famous such cases are probably the $\mathcal{N} = 2$ SCFTs found by Minahan and Nemeschansky in [1,2]. These theories have E_n global symmetries, and their Seiberg-Witten curves are known, along with the exact scaling dimensions of various operators. Additionally, these theories are strongly coupled (as one can read off from the SW curve), and have no known weakly coupled UV descriptions. Some related examples were found in [3,4].

The number of “non-Lagrangian” CFTs saw a dramatic boost with Gaiotto’s discovery of T_N theories in [5]. These $\mathcal{N} = 2$ SCFTs come from M5-branes wrapping a thrice-punctured Riemann surface. The resulting theories have an $SU(N)^3$ global symmetry and do not have any known weakly coupled UV avatars. Despite this lack of information, we do (as in the E_n SCFTs) know a surprising amount about these theories, including many of the gauge-invariant operators in the theory, as well as their scaling dimensions and charges under global symmetries.

One reason we know so much about E_n and T_N theories is because these theories have $\mathcal{N} = 2$ SUSY. Naively, theories with less supersymmetry might seem inaccessible to

study. This is, however, not the case. If we start with a theory with $\mathcal{N} = 2$ SUSY and then break to $\mathcal{N} = 1$, we can often still make exact statements. In particular, if we want to know the scaling dimensions of chiral primary operators, the only thing we need access to is information about global symmetries: As long as we break SUSY in such a way that we can track which global symmetries are unbroken, we can use a -maximization [6] to compute the $\mathcal{N} = 1$ superconformal R-symmetry.¹ The dimension Δ of a chiral primary \mathcal{O} is then given by $\Delta(\mathcal{O}) = \frac{3}{2}R(\mathcal{O})$.

$\mathcal{N} = 1$ deformations of T_N theories were first studied in [9]. In that work, the authors gauge the $SU(N)^3$ global symmetry, and then build theories by connecting T_N 's through these gauge groups. SUSY is then broken by adding a mass term for the $\mathcal{N} = 1$ adjoint chiral superfields in the $\mathcal{N} = 2$ vector multiplets. Integrating out the adjoints results in a superpotential which uniquely fixes the R-symmetry, so a -maximization is unnecessary. The original $\mathcal{N} = 2$ theories were shown in [10] to be dual to the $\mathcal{N} = 2$ Maldacena-Nuñez backgrounds [11]. One of the main results of [9] is that when SUSY is broken to $\mathcal{N} = 1$ by integrating out the adjoint chiral superfield, the resulting theories are dual to the $\mathcal{N} = 1$ Maldacena-Nuñez solutions. This result is also hinted at by the ratio of the central charges in the UV and IR, as discussed in [12].

In this work, we continue the program of [9] by studying further $\mathcal{N} = 1$ deformations of T_N theories. In particular, we consider theories which are coupled to $\mathcal{N} = 1$ vector multiplets, but do not have the superpotentials that result from integrating out the adjoint chiral superfields. This is equivalent to taking a limit where the mass of the adjoint is large. In such cases, there are more global symmetries than in the theories of [9], and we will often need to use a -maximization to determine the superconformal R-symmetry. We note that if the resulting theories are good SCFTs, they should then be thought of as belonging to the family of SCFTs that come from six-dimensional $(2,0)$ -theories descending from M5-branes wrapping a Riemann surface.

The theories we consider in this paper involve T_N theories with gauged global symmetries connected by bifundamental $\mathcal{N} = 1$ chiral superfields. The requisite building blocks are illustrated in Figure 1. For simplicity, in this work all of our bifundamentals will have

¹ This is the philosophy adopted in [7,8], where such computations were performed for $\mathcal{N} = 1$ deformations of $\mathcal{N} = 2$ E_n SCFTs.

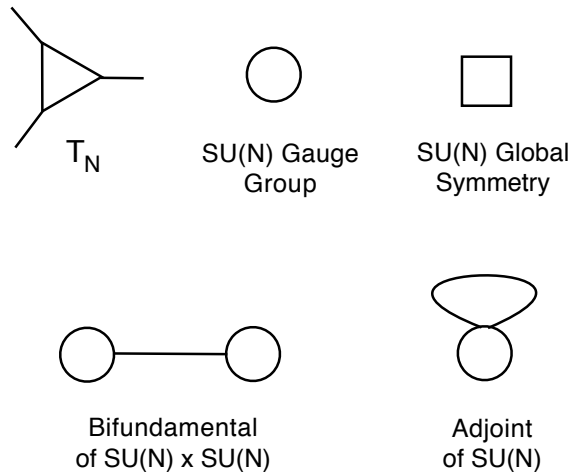


Figure 1: *The basic building blocks of our theories.*

the same matter content as an $\mathcal{N} = 2$ hypermultiplet. We will additionally consider matter fields which are adjoints, since these will show up in theories which are the putative endpoints of certain RG flows.

Our main question is whether the theories we consider are consistent, dynamically realized SCFTs. Of course, in the absence of some kind of duality, it is very difficult to determine whether or not these theories actually achieve an interacting conformal phase. However, we provide a series of checks that some of the theories we consider are in fact new $\mathcal{N} = 1$ SCFTs without Lagrangian descriptions. We use two main criteria: First, we check that the gauge-invariant operators have dimensions Δ that obey the unitarity bound $\Delta \geq 1$. Second, we check that when we deform theories by a relevant operator, or give a vev to a field, the resulting flow does not violate the a -theorem (recently proved in [13]).

We find that when we connect T_N blocks with only hypermultiplets and do not turn on a superpotential, these theories appear to be good SCFTs, in the sense described above. In particular, there are no unitarity bound violations, and when we turn off the superpotential of [9], the resulting flows are consistent with the a -theorem. In contrast, it appears that whenever we add adjoints, the resulting theories do not appear to be consistent SCFTs, since we see flows which are inconsistent with the a -theorem.

The rest of this paper is structured as follows. In Section 2, we review some relevant facts about $\mathcal{N} = 1$ and $\mathcal{N} = 2$ SCFTs and briefly describe our setup. Section 3 describes our flagship family of theories, in which T_N blocks are connected via hypermultiplets. We note that some of these theories require a -maximization, which we perform. In Section

4 we describe some other possible $\mathcal{N} = 1$ setups involving T_N blocks; these theories will show up when we analyze the flows from the theories of Section 3. In Section 5 we describe these flows, note that some of them appear to violate the a -theorem, and discuss possible resolutions. Finally in Section 6, we briefly conclude.

2. Review and Setup

In this section we review the basics of T_N theories from both an $\mathcal{N} = 1$ and $\mathcal{N} = 2$ perspective, and describe the techniques necessary to explore the $\mathcal{N} = 1$ deformations of these theories.

2.1. Review of T_N Technology

T_N theories were discovered by Gaiotto [5] via S-duality. They describe the low-energy four-dimensional $\mathcal{N} = 2$ theories coming from N M5-branes wrapping a thrice-punctured Riemann surface. For $N \geq 3$, these theories have no known weakly coupled Lagrangian descriptions. Although such a fundamental description is lacking, we still have access to a relatively large amount of information, such as global symmetries and operator dimensions. For our purposes here, this information will turn out to be exactly what we need.

The global symmetries of a T_N theory are $SU(2)_R \times U(1)_R \times SU(N)^3$. The Coulomb branch is parameterized by operators $u_k^{(i)}$, with $k = 3, \dots, N$ and $i = 1, \dots, (k-2)$. These operators have scaling dimension $\Delta(u_k^{(i)}) = k$. The Higgs branch is described by dimension-two operators μ_α , for $\alpha = 1, \dots, 3$. Each μ_α transforms in the adjoint representation of one of the $SU(N)$ global symmetries. There are also dimension $(N-1)$ operators Q_{ijk} and \tilde{Q}^{ijk} that are in the trifundamental² $(\mathbf{N}, \mathbf{N}, \mathbf{N})$ and $(\overline{\mathbf{N}}, \overline{\mathbf{N}}, \overline{\mathbf{N}})$ of $SU(N)^3$.

It is convenient to single out the two abelian symmetries inside the $SU(2) \times U(1)$ R-symmetry. We will use I_3 to denote the $U(1)$ generated by $T^3 \subset SU(2)$, and the other $U(1)$ by $R_{\mathcal{N}=2}$. To fix notation, recall that for free vectors and hypers these symmetries have charges as follows:

$$\begin{array}{cccc}
R_{\mathcal{N}=2} \setminus I_3 & \frac{1}{2} & 0 & -\frac{1}{2} \\
0 & & A_\mu & \\
1 & \lambda & & \lambda' \\
2 & & \phi &
\end{array}
\qquad
\begin{array}{cccc}
R_{\mathcal{N}=2} \setminus I_3 & \frac{1}{2} & 0 & -\frac{1}{2} \\
-1 & & \psi & \\
0 & Q & & \tilde{Q}^\dagger \\
1 & & \tilde{\psi}^\dagger &
\end{array}
\tag{2.1}$$

² For an interesting recent discussion of $\mathcal{N} = 2$ theories with trifundamentals, see [14].

For a T_N , the nonzero 't Hooft anomalies for these two global symmetries are [10]:

$$\begin{aligned}\mathrm{Tr}_{T_N} R_{\mathcal{N}=2}^3 &= \mathrm{Tr}_{T_N} R_{\mathcal{N}=2} = 2 + N - 3N^2 \\ \mathrm{Tr}_{T_N} R_{\mathcal{N}=2} I_3^2 &= \frac{1}{12} (6 - N - 9N^2 + 4N^3).\end{aligned}\tag{2.2}$$

Similarly, we can compute the central charges a and c :

$$\begin{aligned}a_{T_N} &= \frac{N^3}{6} - \frac{5N^2}{16} - \frac{N}{16} + \frac{5}{24} \\ c_{T_N} &= \frac{N^3}{6} - \frac{N^2}{4} - \frac{N}{12} + \frac{1}{6}.\end{aligned}\tag{2.3}$$

We will also need to use the fact that if we gauge one of the $SU(N)$ global symmetries of the T_N ,

$$\mathrm{Tr}_{T_N} R_{\mathcal{N}=2} T^a T^b = -N \delta^{ab},\tag{2.4}$$

where T^a is a generator of the gauged symmetry. For a derivation, see [9] or [10].

The purpose of the present paper is to extend the program of [9] by studying the $\mathcal{N} = 1$ properties of T_N theories. To this end, we will make frequent use of various linear combinations of $R_{\mathcal{N}=2}$ and I_3 . One particularly useful choice is

$$\begin{aligned}R_{\mathcal{N}=1} &= \frac{1}{3} R_{\mathcal{N}=2} + \frac{4}{3} I_3 \\ J &= R_{\mathcal{N}=2} - 2I_3.\end{aligned}\tag{2.5}$$

By checking the charge assignments in (2.1), one can see that J is a non-R symmetry. $R_{\mathcal{N}=1}$ is the superconformal R-symmetry for a free $\mathcal{N} = 1$ theory.

2.2. Breaking to $\mathcal{N} = 1$

In this paper, we will be working with T_N 's in theories which have only $\mathcal{N} = 1$ SUSY. The theories we consider will have at least one T_N for which we gauge part of the global symmetry group, and take the gauge boson to be part of an $\mathcal{N} = 1$ vector multiplet. One way to arrive at such a theory is to start with an $\mathcal{N} = 2$ vector multiplet, and give a mass to the associated $\mathcal{N} = 1$ adjoint chiral superfield Φ via a superpotential $W = m \mathrm{Tr} \Phi^2$. The R-symmetry preserved by this deformation is

$$R_0 = R_{\mathcal{N}=1} + \frac{1}{6} J = \frac{1}{2} R_{\mathcal{N}=2} + I_3.\tag{2.6}$$

In the case where the vector multiplet is coupled to a hypermultiplet $Q \oplus \tilde{Q}$, R_0 is the R-symmetry that gives charge 1/2 to both Q and \tilde{Q} , making the superpotential $W = \frac{1}{m}(Q\tilde{Q})^2$ marginal.

The main question we wish to answer here is whether or not there exist good (consistent, dynamically realized) SCFTs in the absence of such a superpotential term. One way to achieve this would be to take m large, in which case the superpotential vanishes. Alternately, one could just consider the theory as coupled only to $\mathcal{N} = 1$ vector multiplets from the start. For our purposes here we will simply turn off this superpotential and then check that the theory satisfies a reasonable set of consistency conditions.

In practice, turning off this superpotential means that the theory can have additional $U(1)$ global symmetries. For example, in a theory with only vector and hypermultiplets, the $U(1)$ denoted by J in (2.5) is no longer explicitly broken by the superpotential. This is just the axial $U(1)$ that comes with any $Q \oplus \tilde{Q}$. Such a symmetry is forbidden by $\mathcal{N} = 2$ SUSY, but is of course allowed in $\mathcal{N} = 1$ theories. When we consider T_N blocks in $\mathcal{N} = 1$ theories, each T_N will similarly come equipped with an “axial” J , whose action on the T_N is given by the linear combination of $U(1)$ ’s in (2.5).

2.3. *a*-maximization

Since we will generically not be able to use symmetries to uniquely determine the superconformal R-symmetry, we will need to use *a*-maximization [6], which we now review. In general, any putative R-symmetry R_{trial} will have the form

$$R_{trial} = R_0 + \sum_I s_I F_I, \quad (2.7)$$

where I runs over all $U(1)$ global symmetries F_I , and the s_I are as yet undetermined coefficients. These s_I are uniquely specified by maximizing the function

$$a_{trial}(s_I) = 3\text{Tr}R_{trial}^3 - \text{Tr}R_{trial}. \quad (2.8)$$

As first noted in [6], global symmetries F_I which have $\text{Tr}F_I = 0$ will never mix with the R-symmetry; this follows from the extremum condition $9\text{Tr}R^2F_I = \text{Tr}F_I$, which means that when $\text{Tr}F_I = 0$ the R-symmetry can always be taken to commute with F_I . The most prominent example of such a symmetry is the baryonic $U(1)$ in SQCD. Thus we will often refer to such $U(1)$ ’s as “baryonic.”

We now record some formulae that will be useful for our computations. The contribution to a_{trial} from each T_N can be computed by writing R_0 and J in terms of $R_{\mathcal{N}=2}$ and I_3 as in (2.5) and (2.6). Taking the coefficient of J to be s , this contribution is

$$a_{trial}^{T_N} = \left(s + \frac{1}{2}\right) \left(3s^2 + 3s - \frac{1}{4}\right) \text{Tr}_{T_N} R_{\mathcal{N}=2}^3 + 9 \left(s + \frac{1}{2}\right) (1 - 2s)^2 \text{Tr}_{T_N} R_{\mathcal{N}=2} I_3^2. \quad (2.9)$$

In this expression, we have used $\text{Tr}_{T_N} R_{\mathcal{N}=2} = \text{Tr}_{T_N} R_{\mathcal{N}=2}^3$. The two 't Hooft anomalies in (2.9) are given in (2.2).

For reference, we additionally note that in $\mathcal{N} = 1$ theories, a hypermultiplet comes with both an axial and a baryonic $U(1)$. For our bifundamental hypermultiplets, the contribution of the axial $U(1)$ to a_{trial} is

$$a_{trial}^{hyper} = 2N^2 \left[3 \left(s - \frac{1}{2}\right)^3 - \left(s - \frac{1}{2}\right) \right], \quad (2.10)$$

where s is the coefficient of F in (2.7). This formula, along with (2.9), will be essentially all we need in order to do computations in our $\mathcal{N} = 1$ theories.

2.4. Building new $\mathcal{N} = 1$ theories

In this paper we will construct theories with three different elements: T_N blocks, $\mathcal{N} = 1$ vector multiplets, and matter chiral superfields. We can depict any such theory by a diagram whose elements are shown in Figure 1. One difference with [9] is that in the present work, we will occasionally include adjoint chiral superfields. We will treat these adjoints as independent $\mathcal{N} = 1$ superfields, and not as part of an $\mathcal{N} = 2$ vector multiplet. This situation will occur naturally in some of the RG flows we study.

The easiest case to consider is one in which all gauge and global symmetry groups are $SU(N)$. We restrict ourselves to such theories in this work. The theories considered here have several types of matter. First, there are bifundamental hypermultiplets $Q \oplus \tilde{Q}$, which transform in the $(\mathbf{N}, \overline{\mathbf{N}}) \oplus (\overline{\mathbf{N}}, \mathbf{N})$ of an $SU(N) \times SU(N)$ gauge group. Additionally, we can have adjoint, fundamental, or antifundamental fields of a given $SU(N)$ gauge group. Finally, we allow T_N 's, as described above.

One particular subclass of such theories is given by a chain of hypermultiplets with a T_N on each end, as in Figure 2. This theory is uniquely specified by the number of hypermultiplets ℓ , and has no superpotential. For convenience, we will denote such theories by the name \mathcal{S}_ℓ . They will be the flagship examples of the present work.

We are now ready to begin checking whether or not \mathcal{S}_ℓ theories are good SCFTs.

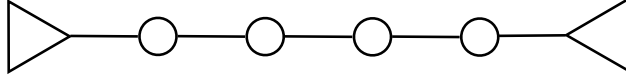


Figure 2: An example of an \mathcal{S}_ℓ theory with $\ell = 3$. The circles are $\mathcal{N} = 1$ vector multiplets.

3. New $\mathcal{N} = 1$ Theories

In this section, we provide some consistency checks that \mathcal{S}_ℓ theories are likely to be interacting $\mathcal{N} = 1$ SCFTs. We begin by doing some simple examples before moving on to the more general case.

3.1. \mathcal{S}_0

Consider a theory with two T_N 's and one vector multiplet. As per our classification, we call this quiver \mathcal{S}_0 . This theory has an anomaly-free R-symmetry R_0 , given in (2.6). Additionally, each T_N has a $U(1)$ global symmetry $J_{1,2}$, for which the anomaly-free linear combination is

$$\mathcal{F} = J_1 - J_2. \quad (3.1)$$

This global $U(1)$ is baryonic, since $\text{Tr } \mathcal{F} = 0$. As discussed above, such a global symmetry will never mix with the R-symmetry. Thus the superconformal R-symmetry here is simply R_0 , and it is straightforward to check that no gauge-invariant operators violate the unitarity bound.

3.2. \mathcal{S}_1

Now consider a theory with two T_N 's, two vectors, and one hyper. In addition to J_1 and J_2 , there is an axial $U(1)$, F , acting only on $Q \oplus \tilde{Q}$. We adopt the normalization $F(Q) = F(\tilde{Q}) = 1$ (note that Q and \tilde{Q} must have the same charge because of charge conjugation symmetry). The hypermultiplet also comes with an anomaly-free baryonic $U(1)$, which we ignore since it does not mix with R_0 . There is an anomaly-free global symmetry given by

$$\mathcal{F} \equiv J_1 + F + J_2. \quad (3.2)$$

Since $\text{Tr } \mathcal{F} \neq 0$, \mathcal{F} can mix with R_0 . Therefore there is a one-parameter family of potential R-symmetries, given by

$$R_{trial}(\alpha) = R_0 + \alpha \mathcal{F}. \quad (3.3)$$

We fix the superconformal R-symmetry by a -maximization. Using (2.9) and (2.10), we find that a_{trial} is maximized at

$$\hat{\alpha}(N) = \frac{-4N + 3N^2 + 4N^3 - 2\sqrt{16 - 16N - 52N^2 + 48N^3 + 33N^4 - 44N^5 + 16N^6}}{6(8 - 11N^2 + 4N^3)}. \quad (3.4)$$

This function is negative and monotonically decreasing for $N \geq 2$, and approaches $-1/6$ at large N . See Figure 3.

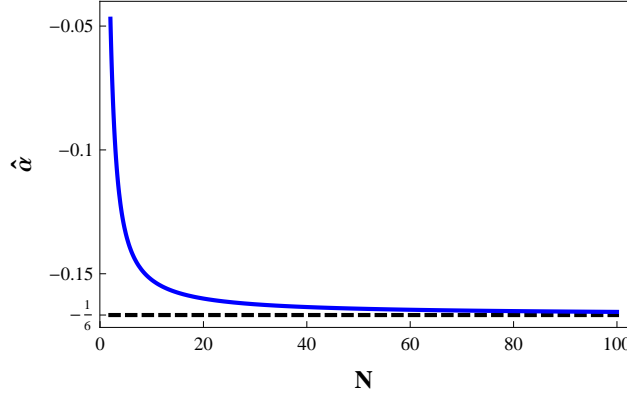


Figure 3: $\hat{\alpha}(N)$ for \mathcal{S}_1 . The dotted line is the asymptotic value $-1/6$.

\mathcal{S}_1 has various gauge-invariant chiral operators constructed from Q , \tilde{Q} , and μ . The R-charges of Q , \tilde{Q} , and μ are

$$R(Q) = R(\tilde{Q}) = \frac{1}{2} + \hat{\alpha}, \quad R(\mu) = 1 - 2\hat{\alpha}. \quad (3.5)$$

Because we have only gauged two of the global $SU(N)$'s, four μ_i remain gauge-invariant. We will denote the remaining two μ operators, which transform in the adjoint of a gauged $SU(N)$, by $\mu_{L,R}$. Thus, the gauge-invariant operators and their superconformal R-charges are schematically

operator	$R(\hat{\alpha})$	
μ_i	$1 - 2\hat{\alpha}$	
$Q\mu_{L,R}\tilde{Q}$	2	
$Q\tilde{Q}$	$1 + 2\hat{\alpha}$	
$QQQ\tilde{Q}$	$2 + 4\hat{\alpha}$	(3.6)

Since $\hat{\alpha} \rightarrow -1/6$ monotonically, no operators violate the unitarity bound $R \geq 2/3$. Thus this theory appears to satisfy one of the necessary criteria for the existence of an SCFT.

We note that at large N , the R-charge of $Q\tilde{Q}$ approaches $2/3$, so this operator becomes free and decouples from the theory. This may prove problematic when trying to find a gravity dual for these theories, if one exists.

3.3. General \mathcal{S}_ℓ

Now we consider the general \mathcal{S}_ℓ theory, which has ℓ bifundamental hypers. We denote the hypermultiplet connecting the i^{th} and $(i+1)^{\text{th}}$ node by $Q_i \oplus \tilde{Q}_i$, where we take the first node to be $i=0$. Each hyper has an axial $U(1)$ symmetry F_i , with charges $F_i(Q_j) = F_i(\tilde{Q}_j) = \delta_{ij}$. The unique anomaly-free non-R $U(1)$ is

$$\mathcal{F}_\ell \equiv J_1 + (-1)^{\ell-1} J_2 + F_o - F_e \quad (3.7)$$

where

$$F_e = \sum_{i=1}^{\lfloor \frac{\ell}{2} \rfloor} F_{2i}, \quad F_o = \sum_{i=1}^{\lfloor \frac{\ell+1}{2} \rfloor} F_{2i-1}. \quad (3.8)$$

Under the reflection symmetry of the quiver, $\mathcal{F}_\ell \rightarrow (-1)^{\ell+1} \mathcal{F}_\ell$. When ℓ is even, $\text{Tr} \mathcal{F}_\ell = 0$, and this $U(1)$ will not mix with the R-symmetry. Thus, for even ℓ , the superconformal R-symmetry is R_0 . When ℓ is odd, the candidate R-symmetry is $R_{\text{trial}} = R_0 + \alpha \mathcal{F}_\ell$. a -maximization yields

$$\hat{\alpha}(\ell, N) = \frac{A - \sqrt{B}}{C}, \quad (3.9)$$

where

$$\begin{aligned} A &= -4N + 3\ell N^2 + 4N^3 \\ B &= 64 - 64N - 208N^2 - 24(\ell - 9)N^3 + 3(3\ell^2 + 41)N^4 + 8(3\ell - 25)N^5 + 64N^6 \\ C &= 6(8 - 11N^2 + 4N^3). \end{aligned} \quad (3.10)$$

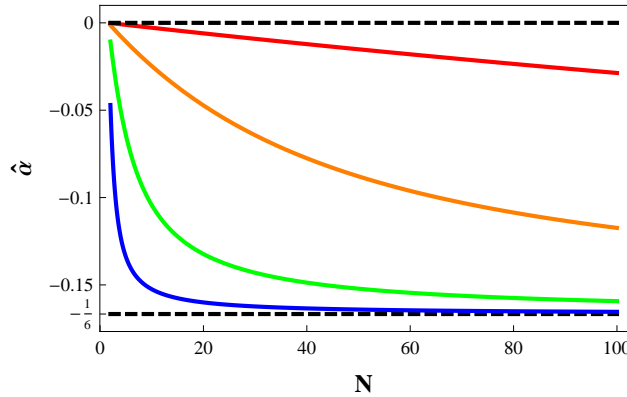


Figure 4: $\hat{\alpha}(\ell, N)$ for various \mathcal{S}_ℓ as a function of N .

The colors correspond to $\ell = 1$ (blue, bottom), 11, 101, and 1001 (red, top).

We plot $\hat{\alpha}(\ell, N)$ in Figure 4. For all values of ℓ and N , $\hat{\alpha}$ is negative. At fixed ℓ , $\hat{\alpha}$ is a monotonically decreasing function of N that asymptotes to $-1/6$. At fixed N , $\hat{\alpha}$ monotonically increases and approaches zero. In this case, the R-charges of operators in theories with odd and even ℓ become the same.

We can now compute the dimensions of all the chiral primary operators in the theory. For even ℓ , all $R(Q) = R(\tilde{Q}) = 1/2$, and $R(\mu) = 1$ for any Q, \tilde{Q} , and μ . The only relevant operators in this theory are $Q\tilde{Q}$. The other gauge-invariant operators, $\mu Q\tilde{Q}$ and $Q\tilde{Q}Q\tilde{Q}$, are marginal.

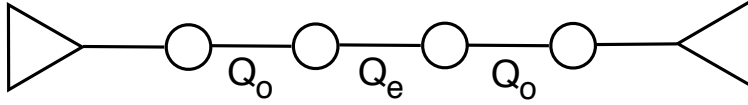


Figure 5: S_3 , with Q_o and Q_e marked.

When ℓ is odd, the dimension of a hypermultiplet depends on its position in the quiver. This follows from the different signs of F_o and F_e in (3.7). We will denote bifundamentals which are to the left of odd numbered nodes by Q_o, \tilde{Q}_o , and the other bifundamentals by Q_e, \tilde{Q}_e . See Figure 5. From (3.7), we see that these fields have R-charges

$$R(Q_o) = R(\tilde{Q}_o) = \frac{1}{2} + \hat{\alpha}, \quad R(Q_e) = R(\tilde{Q}_e) = \frac{1}{2} - \hat{\alpha}. \quad (3.11)$$

The simplest gauge-invariant operators we can form from these operators are quadratic. Such fields have R-charges

$$R(Q_o\tilde{Q}_o) = 1 + 2\hat{\alpha}, \quad R(Q_e\tilde{Q}_e) = 1 - 2\hat{\alpha}. \quad (3.12)$$

We can also make quartic operators, with R-charges

$$R((Q_o\tilde{Q}_o)^2) = 2 + 4\hat{\alpha}, \quad R((Q_e\tilde{Q}_e)^2) = 2 - 4\hat{\alpha}, \quad R(Q_o\tilde{Q}_oQ_e\tilde{Q}_e) = 2. \quad (3.13)$$

Finally, we note that the gauge-invariant operators involving μ_i have R-charge

$$R(\mu_i Q_o\tilde{Q}_o) = 2. \quad (3.14)$$

There are no operators of the form $\mu_i Q_e\tilde{Q}_e$ in theories with odd ℓ .

At fixed ℓ , $\hat{\alpha}$ is a monotonic function that approaches $-1/6$ at large N . In this case, it is easy to see that only the quadratic operators and $(Q_o\tilde{Q}_o)^2$ can be relevant deformations of the theory. When we fix N but take ℓ large, $\hat{\alpha} \rightarrow 0$. Thus the quadratic operators are relevant and the quartic operators are all marginal. Note that for both even and odd ℓ , all gauge-invariant operators respect the unitarity bound for any values of ℓ and N .

4. Other $\mathcal{N} = 1$ theories

We now move on to consider other $\mathcal{N} = 1$ theories which are similar to the \mathcal{S}_ℓ theories constructed above.

4.1. Theories with one T_N .

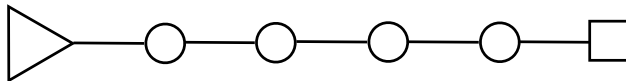


Figure 6: The theory \mathcal{S}_3^{\square} .

In this section, we study \mathcal{S}_ℓ theories where the right T_N has been replaced by N fundamental hypermultiplets, as in Figure 6. We will name these theories $\mathcal{S}_\ell^{\square}$. Since the T_N 's and N hypermultiplets have the same contribution to the gauge anomaly, R_0 is still anomaly-free. The total number of hypermultiplets is $\ell + 1$, where ℓ of the hypers are bifundamental. This theory has an anomaly-free non-R $U(1)$ global symmetry

$$\mathcal{F}_\ell = J - \sum_i^\ell (-1)^i F_i + (-1)^\ell F_f \quad (4.1)$$

where J acts on the T_N and F_f acts on the fundamental hypermultiplets with charge 1. The trial R-symmetry is

$$R_{trial} = R_0 + \alpha \mathcal{F}_\ell. \quad (4.2)$$

Once again, we can determine $\hat{\alpha}$ by a -maximization. We plot the result in Figure 7. At fixed ℓ , $\hat{\alpha}$ is a monotonically decreasing function of N that approaches $-\frac{1}{6}$. At $N = 2$, it vanishes when ℓ is even. This follows from the fact that T_2 is a theory of two hypermultiplets. In this case, the quiver has a reflection symmetry which switches the T_2 on the left with the two fundamentals on the right. When ℓ is even, \mathcal{F}_ℓ is odd under reflection while R_0 is even. Thus the two $U(1)$'s cannot mix. In general, $\hat{\alpha}$ is bounded above by zero. The behavior of the gauge-invariant operators constructed from the hypermultiplets is the same as in the \mathcal{S}_ℓ theories. As a consequence, they do not violate the unitarity bound.

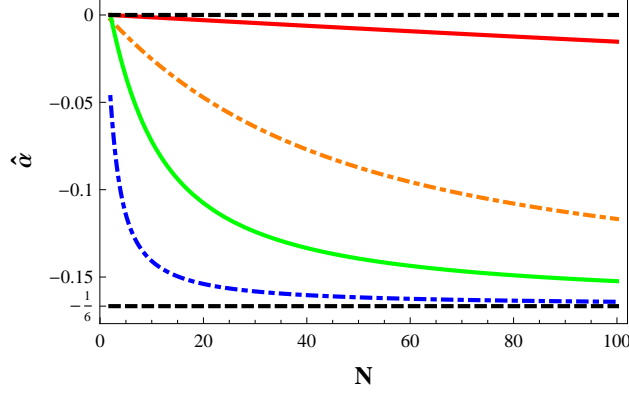


Figure 7: $\hat{\alpha}(\ell, N)$ for \mathcal{S}_ℓ^\square at $\ell = 1$ (dashed blue, bottom), 10, 51, 1000 (solid red, top).

4.2. Ending with adjoints

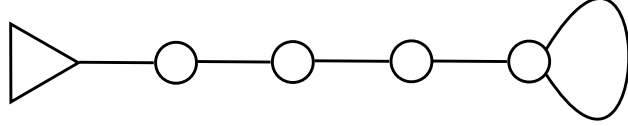


Figure 8: The theory \mathcal{S}_3° .

We now add adjoint matter to the \mathcal{S}_ℓ theories. The simplest thing we can do is substitute the T_N at the right end of the quiver with a chiral superfield X in the adjoint of $SU(N)$, as in Figure 8. We will call these theories \mathcal{S}_ℓ° . R_0 is anomaly-free when $R_0(X) = \frac{1}{2}$. Additionally, X comes with a $U(1)$, F_a , which we normalize as $F_a(X) = 1$. The theory then has an anomaly-free global $U(1)$ given by

$$\mathcal{F} = J_1 - \sum_{i=1}^{\ell} (-1)^i F_i + (-1)^\ell F_a. \quad (4.3)$$

The trial R-symmetry can be written as

$$R_{trial} = R_0 + \alpha \mathcal{F}, \quad (4.4)$$

so the charge of X and its contribution to the trial central charge are

$$\begin{aligned} R_{trial}(X) &= \frac{1}{2} + (-1)^\ell \alpha \\ a_{trial}^{X,\circ} &= (N^2 - 1) \left[3 \left(-\frac{1}{2} + (-1)^\ell \alpha \right)^3 - \left(-\frac{1}{2} + (-1)^\ell \alpha \right) \right]. \end{aligned} \quad (4.5)$$

Maximizing a yields an answer very similar to that of the \mathcal{S}_ℓ theories, although for simplicity we do not record the full answer here. Instead, we note that $\hat{\alpha}$ is a monotonically decreasing function of N that approaches $-\frac{1}{6}$ in $N \gg \ell$ limit. When $\ell \gg N$, $\hat{\alpha}$ approaches zero; this is consistent with the contribution of the hypermultiplets dominating over those of the T_N . In Figure 9, we plot $\hat{\alpha}$ as a function N at different values of ℓ .

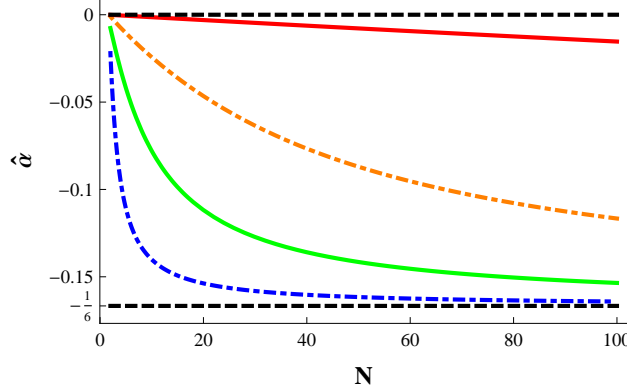


Figure 9: $\hat{\alpha}(\ell, N)$ for \mathcal{S}_ℓ° at $\ell = 1$ (dashed blue, bottom), 10, 51, 1000 (solid red, top).

The gauge-invariant operators in this theory do not violate the unitarity bound. This is easy to see for operators not involving X , since $\hat{\alpha}$ is again a monotonic function with the same asymptotic behavior as in the \mathcal{S}_ℓ theories. The lowest-dimension operators involving X are $\text{Tr} X^2$ and $Q_\ell X \tilde{Q}_\ell$, which have R-charges

$$R(X^2) = 1 + 2(-1)^\ell \hat{\alpha}, \quad R(Q_\ell X \tilde{Q}_\ell) = \frac{3}{2} - (-1)^\ell \hat{\alpha}. \quad (4.6)$$

Since $-1/6 < \hat{\alpha} < 0$, these operators are always above the unitarity bound, and relevant.

4.3. Inserting adjoints in the middle

Now we consider a different theory, with the adjoint chiral field X attached to the k^{th} $SU(N)$ gauge group of the \mathcal{S}_ℓ theories (we let $k = 0$ be the first node, the one attached to the left T_N). We will name these theories $\mathcal{S}_{\ell,k}$. See Figure 10. To preserve the R-anomaly, $R_0(X) = 1$. Here the adjoint leads to an additional anomaly-free global $U(1)$, since we have increased the number of fields but not the number of constraints on their charges. Taking $F_a(X) = 1$, a convenient choice for these global $U(1)$'s is

$$\begin{aligned} \mathcal{F}^L &= J_1 - \sum_{i=1}^{k-1} (-1)^i F_i - (-1)^k F_a \\ \mathcal{F}^R &= J_2 + \sum_{i=k}^{\ell} (-1)^{\ell+i} F_i - (-1)^{\ell+k} F_a. \end{aligned} \quad (4.7)$$

Note that \mathcal{F}_ℓ in (3.7) can be written $\mathcal{F}_\ell = \mathcal{F}^L - (-1)^\ell \mathcal{F}^R$.

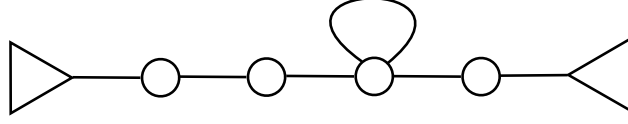


Figure 10: The theory $\mathcal{S}_{3,2}$.

From the trial R-symmetry

$$R_{trial} = R_0 + \alpha_L \mathcal{F}^L + \alpha_R \mathcal{F}^R, \quad (4.8)$$

we observe that we need to a -maximize over two variables. The charge of X and its contribution to a_{trial} are

$$\begin{aligned} R_{trial}(X) &= 1 - (-1)^k (\alpha_L + (-1)^\ell \alpha_R), \\ a_{trial}^{X,\ell,k} &= -(-1)^k (N^2 - 1) (3(\alpha_L + (-1)^\ell \alpha_R)^3 - (\alpha_L + (-1)^\ell \alpha_R)). \end{aligned} \quad (4.9)$$

After a -maximization, we find the following:

1. At fixed N , the value of $(\hat{\alpha}_L, \hat{\alpha}_R)$ at (l, k) is equal to $(\hat{\alpha}_R, \hat{\alpha}_L)$ at $(l, l - k + 2)$. This follows from the reflection symmetry of the quiver, which switches the two T_N 's and maps the i^{th} gauge group to $\ell - i + 2$. The adjoint which was attached to node k moves to node $l - k + 2$. Under this reflection, $\hat{\alpha}_L \leftrightarrow \hat{\alpha}_R$.
2. $\hat{\alpha}_{L,R}$ are monotonically decreasing functions of N that approach $-\frac{1}{6}$ in the $N \gg \ell$ limit. They are bounded above by zero. The R-charge of X ranges from two-thirds to four-thirds. The former occurs in the large N limit of the quiver with odd values of k but even values of ℓ , whereas the latter occurs in the large N limit of the quivers with even values of k and ℓ .
3. In the $\ell \gg N$ limit, we observe various phenomena depending on whether k is small or large. When $k \ll \ell$, the pair $(\hat{\alpha}_L, \hat{\alpha}_R)$ approaches $(-\frac{1}{6}, 0)$ when N is subsequently taken large. The left side behaves as the theory in Figure 8 with small ℓ while the right side behaves as this theory with $\ell \gg N$. In the case when $k \sim \frac{\ell}{2}$, $(\hat{\alpha}_L, \hat{\alpha}_R)$ approaches $(0, 0)$ since each side is like the theory in Figure 8 with $\ell \gg N$.
4. For all values of N, l , and k , the R-charge of X is above the unitarity bound. This implies that all gauge-invariant operators constructed from X are above the unitarity bound. Since the operators from the hypermultiplets are also well behaved, this theory obeys the unitary bound. However, in the next section, we will see that these theories appear as the natural endpoints of flows which violate the a -theorem. This suggests that these adjoint theories do not exist as interacting SCFTs.

5. Flows

In this section we describe some possible flows between our new $\mathcal{N} = 1$ theories. We consider flows driven both by deformations by relevant operators and by Higgsing, focusing in particular on the \mathcal{S}_ℓ theories. We begin by considering linear deformations of these theories, and then proceed upwards to the quartics.

5.1. Linear Deformations

The only possible gauge-invariant linear deformation of an \mathcal{S}_ℓ theory is μ_i , which transforms in the adjoint of the remaining global $SU(N)$ groups. Since μ is a composite field (albeit one whose description in terms of fundamental fields we don't know), this deformation does not break SUSY. When ℓ is even, $R(\mu) = 2$ but when ℓ is odd, $R(\mu) < 2$ and is relevant. The theory resulting from this deformation is mysterious, and we do not at present have a good understanding of the endpoint of this flow.

5.2. Quadratic Deformations

The mesons $Q_o\tilde{Q}_o$ and $Q_e\tilde{Q}_e$ are relevant for all ℓ and N . These operators are mass terms for the quarks. When they are turned on, we can integrate out the associated hypermultiplets. The resulting theory consists of two disconnected parts, each ending with an $SU(N)$ gauge group with $N_f = N_c = N$ flavors. As is well-known, this group then has a quantum constraint on the moduli space, $\det M - B\tilde{B} = \Lambda^{2N_c}$. Even if the adjoining group with $N_f = 2N_c = 2N$ becomes strong, this holomorphic, exact constraint is valid. Thus, we expect that at a generic point on this moduli space, the mesons M will get vevs and break the adjacent $SU(N)$ group to $U(1)^{N-1}$. Although the gauge group will flow free in the IR, the gauge-invariant fields will be part of a complicated sigma model, whose full quantum description is difficult to analyze.

One special point we can analyze is when $\langle M \rangle = 0$. Here, the baryons get vevs, but this has no effect on the adjacent gauge groups. When the mesons for each of the disconnected parts have zero vev, the theory splits into two disconnected \mathcal{S}_ℓ° theories. In particular, if we integrate out the bifundamental between nodes $k-1$ and k , the IR theory is then $\mathcal{S}_{k-2}^\circ \oplus \mathcal{S}_{\ell-k-1}^\circ$, at this special point in moduli space.

We can now compute the difference in central charges and check agreement with the recently proven a -theorem [13]. In Figure 11, we plot $a_{UV} - a_{IR}$ for $\ell = 100$ at various values of k . This behavior is generic for even values of ℓ . It is easy to understand the

violation at large N . In this limit, the central charge is dominated by the T_N , whose contribution to a scales as N^3 . As discussed in Section 3, the superconformal R-symmetry of the \mathcal{S}_ℓ theories is R_0 when ℓ is even. In the T_N sector of this theory, the R-symmetry is $R_0 - \frac{1}{6}J$. Thus the T_N contributions of $\mathcal{S}_{k-2}^\circ \oplus \mathcal{S}_{\ell-k-1}^\circ$ will always be greater than \mathcal{S}_ℓ .

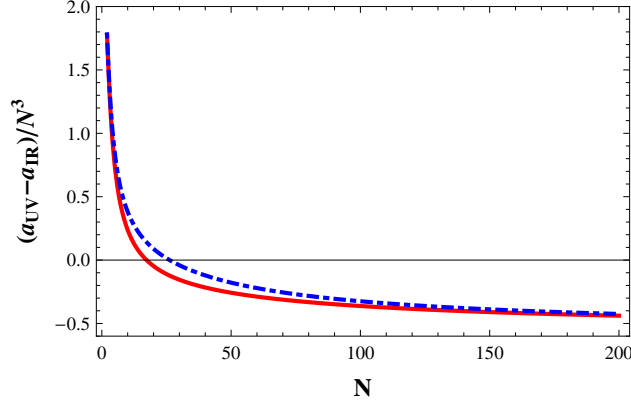


Figure 11: Difference of central charges between \mathcal{S}_ℓ and $\mathcal{S}_{k-2}^\circ \oplus \mathcal{S}_{\ell-k-1}^\circ$ at $\ell = 100$.

The colors correspond to $k = 3$ (red, solid) and 53 (blue, dashed).

In Figure 12, we plot the difference in central charges for $\ell = 151$ as a representative example of the case with odd ℓ . At large N , the difference is always positive. This follows from the fact that the R-symmetry acting on the T_N is always $R_0 - \frac{1}{6}J$ for both \mathcal{S}_ℓ and \mathcal{S}_ℓ° when ℓ is odd. Thus the T_N contributions cancel and any difference must come from the hypermultiplets and vector multiplets. These do not violate the a -theorem. At finite N , we observe a -theorem violations when $k \ll \ell$, i.e. when we integrate out fields close to the T_N .

We can also integrate out bifundamentals in the \mathcal{S}_ℓ^\square theory and obtain an \mathcal{S}_k° theory, along with a quiver of vectors and hypermultiplets with an adjoint. We observe a -theorem violations at large N when $k \ll \ell$. We have not included the relevant plots here, as they are quite similar to the ones already discussed. We conclude that the \mathcal{S}_k° theories cannot be the endpoint of flows from either the \mathcal{S}_ℓ or \mathcal{S}_ℓ^\square theories. Since we have no dynamical way to obtain the \mathcal{S}_k° theories, it is reasonable to conclude that they do not exist.

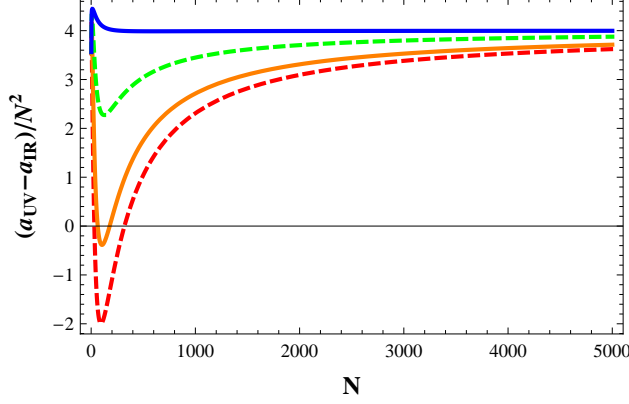


Figure 12: Difference of central charges between \mathcal{S}_ℓ and $\mathcal{S}_{k-2}^\circ \oplus \mathcal{S}_{\ell-k-1}^\circ$ at $\ell = 151$.

The color scheme is $k = 3$ (dashed red, bottom), 12, 33, 60, (blue, top).

5.3. Cubic Deformations

There is only one possible cubic deformation, $\mu Q\tilde{Q}$. This operator has R-charge 2 for both even and odd ℓ and thus is not a relevant deformation. Since we can add it to the theory without breaking any global symmetries, it is an exactly marginal operator and leads only to movement along the conformal manifold.

5.4. Quartic Deformations

For even values of ℓ , all quartic operators are marginal. Since such operators do not break any global symmetries, they simply move the theory along the conformal manifold. We thus consider only odd values of ℓ for the rest of this subsection.

For odd ℓ , the quartic operators $(Q_o\tilde{Q}_o)^2$ are relevant. Adding this operator to the superpotential breaks the global $U(1) \mathcal{F}$ but preserves R_0 . Thus the IR R-symmetry is R_0 . With this R-symmetry, every other possible quartic term has R-charge 2, so we can deform the theory by these operators without changing the R-symmetry. This is consistent with the fact that the deformed theory has no non-R global symmetries, so according to [15], all marginal deformations are exactly marginal. This theory is part of the conformal manifold of theories that one would get by breaking from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ by giving a mass to the adjoint chiral superfield inside the $\mathcal{N} = 2$ vector multiplet, i.e. the theories studied in [9]. The resulting difference between the central charges is plotted in Figure 13. One can see that there is no violation of the a -theorem.

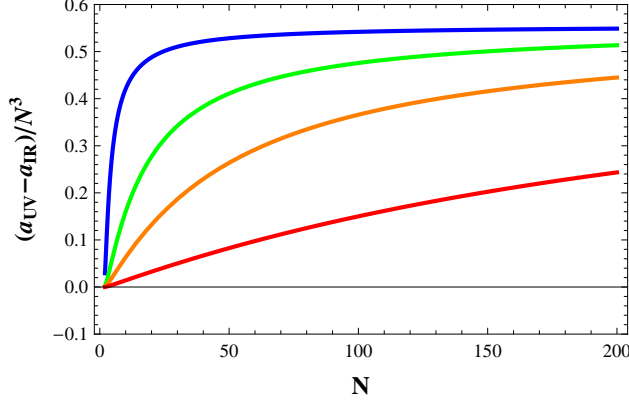


Figure 13: Difference of central charges between \mathcal{S}_ℓ and \mathcal{S}_ℓ with $(Q_o \tilde{Q}_o)^2$ turned on. The colors correspond to $\ell = 1$ (blue, top), 31, 101, and 501 (red, bottom).

The two other kinds of quartic operators are less interesting. The quartic $(Q_e \tilde{Q}_e)^2$ is always irrelevant, and thus drives the flow back to the original SCFT. Finally, $Q_o \tilde{Q}_o Q_e \tilde{Q}_e$ is marginal, and moves the theory along the conformal manifold.

5.5. Higgsing

We now consider giving a vev to a bifundamental Q ,

$$\langle Q \rangle = v \mathbf{1}_{N \times N} \quad (5.1)$$

If Q is a bifundamental of $SU(N)_1 \times SU(N)_2$, this vev breaks $SU(N)_1 \times SU(N)_2 \rightarrow SU(N)_{diag}$. Being on the D-flat moduli space implies that $Q^i Q_j^\dagger - \tilde{Q}^{\dagger i} \tilde{Q}_j = c \delta_j^i$, so we do not need to give a vev to \tilde{Q} to solve the D-terms. However, if we choose to do so, giving a vev to \tilde{Q} just means giving a vev to a singlet of $SU(N)_{diag}$, which does not dramatically affect the resulting theory.

When $SU(N) \times SU(N) \rightarrow SU(N)$, each bifundamental decomposes as $(\mathbf{N}, \bar{\mathbf{N}}) \rightarrow (\mathbf{N}^2 - \mathbf{1}) + \mathbf{1}$. One such adjoint gets eaten, but the remaining adjoint stays in the quiver. The resulting theory is a $\mathcal{S}_{\ell,k}$ theory with $\ell - 1$ nodes and an adjoint chiral superfield attached to one of the nodes. In particular, if we give a vev to a Q between nodes k and $k + 1$ in an \mathcal{S}_ℓ theory, the resulting theory is $\mathcal{S}_{\ell-1,k}$.

In Figure 14, we plot the difference of the central charges of \mathcal{S}_{100} and $\mathcal{S}_{99,k}$ for several values of k . The result is similar to what happens when we integrate out a bifundamental. At large N , there are a -theorem violations. This picture is generic when ℓ is even and independent of k .

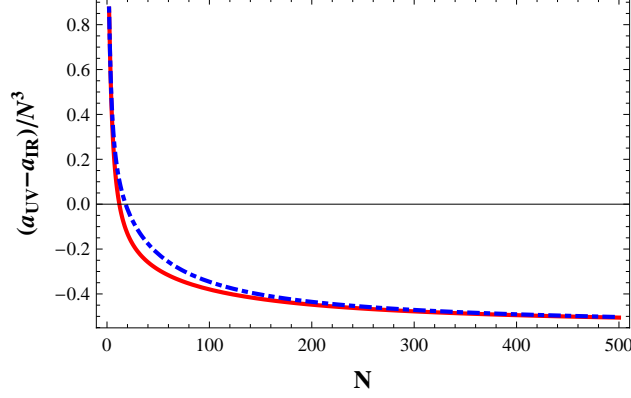


Figure 14: Difference of central charges between \mathcal{S}_ℓ and $\mathcal{S}_{\ell-1,k}$ at $\ell = 100$.

The colors correspond to $k = 4$ (red, bottom) and 50 (dashed blue, top).

When ℓ is odd, the R-symmetry at large N in the T_N sector is the same for both \mathcal{S}_ℓ and $\mathcal{S}_{\ell-1,k}$. In Figure 15, we plot the difference in central charges for $\ell = 101$. Similar to the case where we integrate out a bifundamental, in the large N limit the difference in central charges will come from contributions of the hypermultiplets and vector multiplets. These do not violate the a -theorem. We do observe a -theorem violations at finite N , when $k \ll \ell$.

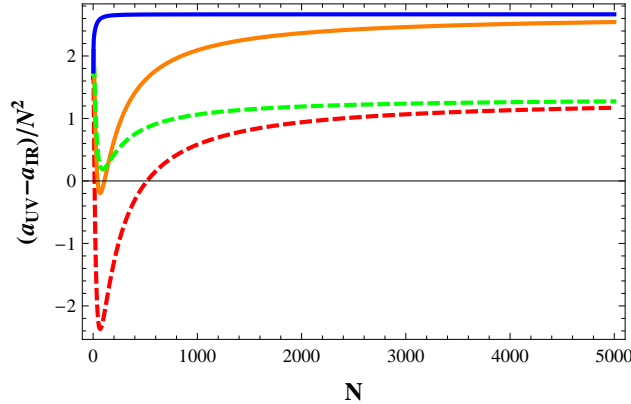


Figure 15: Difference of central charges between \mathcal{S}_ℓ and $\mathcal{S}_{\ell-1,k}$ at $\ell = 101$.

The color scheme is $k = 3$ (dashed red, bottom), $10, 21, 50$ (blue, top).

Because of these a -theorem violations, it appears that the $\mathcal{S}_{\ell,k}$ cannot be the endpoint of flows from \mathcal{S}_ℓ theories. Since we cannot generate them dynamically, we also conclude that they do not exist.

5.6. Discussion

In the previous subsections, we have seen many flows that violate the a -theorem, as well as some that do not. Since the a -theorem has now been proven, such flows cannot exist (and even beforehand, these flows would have seemed suspect). This leaves us with a puzzle: How are we to interpret the theories that show up at the endpoints, both UV and IR, of these flows?

Since both endpoints of the flow are strongly coupled, it is difficult to make definitive statements about whether or not these theories exist. This is generically true for strongly coupled SCFTs, and only in the presence of a duality (which we do not have here) can we hope for a stronger statement about the existence of a conformal phase.

Still, we believe that the evidence presented above indicates that some of the theories we construct lead to good SCFTs while others do not. In particular, the \mathcal{S}_ℓ theories appear to be legitimate new SCFTs. For even ℓ , the \mathcal{S}_ℓ theories are part of the conformal manifold of the $\mathcal{N} = 1$ Sicilian theories described in [9]. The R-symmetry is R_0 , and we do not change this by turning off the superpotential. Thus the even ℓ theories should be interacting SCFTs. From the UV point of view, it is reasonable (though by no means conclusive) to assume that the same is true for odd ℓ as well, even though the odd ℓ theories are no longer on the conformal manifold of the Sicilian theories of [9]. However, the consistent flow from the theories in the present work to those of [9] seems to indicate that these theories are well-behaved, and thus we posit that they exist as good SCFTs. A similar logic applies to the \mathcal{S}_ℓ^\square theories, since we can realize them by integrating out adjoints and subsequently going to the origin of the conformal manifold in the associated $\mathcal{N} = 2$ theory.

The theories with adjoints are more problematic. Since every dynamical realization we have tried in the present work, whether it be through Higgsing or via relevant deformations, leads to flows inconsistent with the a -theorem, we have several options. The first is that these theories simply do not have interacting conformal phases. Another is that there are some accidental symmetries that arise which we cannot see here. In either case, it is true that the theories as presented above are not SCFTs. In the absence of further evidence that they should exist, we therefore conclude that the theories with adjoints do not appear to be part of the landscape of $\mathcal{N} = 1$ SCFTs one can build from T_N blocks.

We thus arrive at the following categorization of the theories considered in this paper.

$$\begin{array}{ll}
\mathcal{S}_\ell & \text{good} \\
\mathcal{S}_\ell^\square & \text{good} \\
\mathcal{S}_\ell^\circ & \text{bad} \\
\mathcal{S}_{\ell,k} & \text{bad}
\end{array} \tag{5.2}$$

Since we find evidence that theories with adjoints and no superpotential all appear to be bad, it is worth asking why this is so. Naively, this is perhaps not surprising – starting with an $\mathcal{N} = 2$ SCFT, one would not generically expect to be able to turn off the $QX\tilde{Q}$ terms without doing violence to the theory. For example, it may be that the gravity duals (i.e. the Maldacena-Nuñez solutions) of the $\mathcal{N} = 2$ theories become singular when we try to turn off the $\mathcal{N} = 2$ superpotential. It is far from clear that the theories we consider here should have gravity duals at all, and we plan to come back to this question in future work.

Another natural question to ask is what happens in the IR for the flows which look like they violate the a -theorem. We have provided evidence that the naive endpoints are not good theories, but if they are not the IR CFT, then where does the theory go? This is obviously a very interesting and important question for understanding the dynamics of the theories considered in this paper, and at present we do not have a good understanding of what the right IR phase is. One interesting possibility is that the theory arrives at a mixed phase (as in [16-19]). Another option is that, when we give mass to a bifundamental in an \mathcal{S}_ℓ theory, the entire theory becomes massive. We hope to return to this question in the future.

6. Conclusions

The main goal of this paper has been to take a small step in the exploration of new $\mathcal{N} = 1$ theories that one can build with T_N blocks. It is not *a priori* obvious which such theories lead to interacting SCFTs and which do not. In the simplest case, where T_N 's are connected with hypermultiplets and intervening adjoints, our work indicates that only theories without adjoints are good SCFTs. However, this leaves open many questions and directions for future work.

The first question is whether one can extend the constructions presented here and build new SCFTs with T_N 's and hypermultiplets. For example, one potential avenue towards new SCFTs is to hook together the \mathcal{S}_ℓ theories. The exploration of such theories

is presently underway [20], and the preliminary results suggest that the \mathcal{S}_ℓ theories can be connected in interesting ways, leading to a vast new landscape of $\mathcal{N} = 1$ SCFTs.

Another interesting question is whether the theories presented here have gravity duals. If such dual theories exist, they should fall into the classification of [21], and show up via M5-branes wrapping an appropriate (punctured) Riemann surface in a $(2, 0)$ six-dimensional theory. It would be interesting to determine, for example, the $\mathcal{N} = 1$ version of the story presented in [10].

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